

國立清華大學命題紙

九十一學年度 化工系轉學生招生考試

科目 微積分 科號 063 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

一. 填充題 (共六題, 每題八分, 共 48 分, 請將答案依甲、乙、丙次序作答, 不需演算過程)

1. $\lim_{x \rightarrow \infty} x \ln \left(\frac{x+1}{x-1} \right) =$ 甲

2. Define $F(x) = \int_{\sin x}^{\cos x} e^{t^2+xt} dt$. Then $F'(0) =$ 乙

3. $\int_0^{\frac{\pi}{2}} \frac{dx}{4 \sin x + 3 \cos x} =$ 丙

4. Let P be the tangent plane to the surface $x^2 + 2y^2 + 2z^2 = 14$ at the point $(2, 1, -2)$. Then the equation of $P =$ 丁

5. Let parabola $Q: y = ax^2 + bx + c$ ($a < 0$) pass through the points $(-1, 1)$ and $(1, 1)$. Then the minimum of the area of the region between by the parabola Q and the x -axis = 戊

6. Let R be the region inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$. Then the area of $R =$ 己

二. 計算題 (共 52 分, 必須寫出演算過程)

1. (11%) Prove that $(\cos \theta)^p \leq \cos(p\theta)$ for $0 \leq \theta \leq \pi/2$ and $0 < p < 1$.

2. Let p and q be real constants. Do the followings by Calculus.

(3%) (a) Prove that, if $p > 0$, the equation $x^3 + px + q = 0$ has exactly one real root.

(8%) (b) Prove that, if $4p^3 + 27q^2 < 0$, the equation $x^3 + px + q = 0$ has exactly three distinct real roots.

3. (8%) (a) Determine whether $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges. (State the theorem used and check the conditions.)

(7%) (b) Determine whether $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges. (State the theorem used and check the conditions.)

4. (15%) Find the absolute maximum and absolute minimum values of

$$f(x, y) = x^2 + y^2 - 2x - 4y$$

on the disk R of radius 3 and center $(0, 0)$.