國　立　清　華　大　學　命　題　紙

九十一學年度 化　學 系轉學生招生考試

科目 微　積　分 科號 041 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

一. 填充題（共六題，每題八分，共 48 分，請將答案依甲、乙、丙次序作答，不需演算過程）

1. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}=$ 甲.

2. Define $F(x)=\int_{\sin x}^{\cos x} e^{t^2+xt}dt$. Then $F'(0)=$ 乙.

3. $\int_0^{\pi}\frac{dx}{2+\cos x}=$ 丙.

4. Let D be the solid bounded by the planes

$$x=0,\ y=0,\ z=0,\ \text{and}\ x+y+z=1.$$

Then the triple integral $\iiint_D z dV=$ 丁.

5. Let parabola $Q: y=ax^2+bx+c\ (a<0)$ pass through the points $(-1,1)$ and $(1,1)$. Then the minimum of the area of the region between by the parabola Q and the x-axis = 戊.

6. Let R be the cardioid $r=1-\cos\theta,\ 0\le\theta\le 2\pi$. Then the length of $R=$ 己.

二. 計算題（共 52 分，必須寫出演算過程）

1. (11%) Prove that $\sin x > x-\frac{x^3}{6}$ for all $x>0$.

2. Let p and q be real constants. Do the followings by Calculus.

(3%) (a) Prove that, if $p>0$, the equation $x^3+px+q=0$ has exactly one real root.

(8%) (b) Prove that, if $4p^3+27q^2<0$, the equation $x^3+px+q=0$ has exactly three distinct real roots.

3. (15%) Determine whether $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ converges for constant $p>0$. (State the theorem used and check the conditions.)

4. (15%) Find the point(s) p on the set $\{z=xy-1\}\cap\{x+y+z+1=0\}$ that is nearest the origin. Also find the distance between the origin and the point p.