

九十一學年度 八系聯招系轉學生招生考試

科目 微積分 科號 003 共 / 頁第 / 頁 *請在試卷【答案卷】內作答

I. 填充題 (共三題, 每題八分, 請將答案依甲, 乙, 丙次序作答, 不需演算過程)

(1). Find $\lim_{x \rightarrow +\infty} \{(x^3 - 2x^2 + 1)^{1/3} - x\}$. Ans. 甲

(2). Find the domain of convergence of $\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} (x+1)^n$. (including the end points) Ans. 乙

(3). Evaluate the surface integral $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} \, d\sigma$, where S is the hemisphere $\{(x, y, z) | x^2 + y^2 + z^2 = 1, z \geq 0\}$, oriented upward, and $\vec{F}(x, y, z) = (x^2 \sin z, x, (1+z)e^{xy})$. Ans. 丙

II. 計算與證明題 (共七題, 必須寫出演算證明過程)

(1). (12 points) Let $f(x, y) = \frac{x^2 y + y^4}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Let $\vec{u} = (a, b)$ be a unit vector. Find the directional derivative $D_{\vec{u}} f(0, 0)$. Is f differentiable at $(0, 0)$? Give your reasons.

(2). (12 points) Find the critical points of $f(x, y) = x^3 + y^2 - 27x + 4y + 1$ and determine whether it is a maximum, minimum or saddle point.

(3). (12 points) Evaluate the integral $\iint_D e^{\frac{x}{x-2y}} \, dx \, dy$, where D is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(-1, -1)$ and $(-2, -2)$.

(4). (10 points) Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a bounded function, i.e., there is a $M > 0$ such that $|f(x)| \leq M$ for all $x \in (-1, 1)$. Define $g(x) = xf(x)$. Is g differentiable at 0? Give your reasons.

(5). (10 points) Evaluate $\int_2^{10} \frac{x+1}{x\sqrt{x-1}} \, dx$.

(6). (10 points) Find the extreme values of $f(x, y, z) = xy + z^2$ subject to the constraints: $x^2 + y^2 + z^2 = 4$ and $x - y = 0$.

(7). (10 points) Apply Green's theorem to find the area of the region enclosed by the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.