I. 填充題（共六題，每題八分，請將答案依甲、乙、丙....次序作答，不需演算過程）

1. $\lim_{n \to \infty} \left(\dfrac{1}{2} + \dfrac{3}{2^2} + \dfrac{5}{2^3} + \cdots + \dfrac{2n-1}{2^n} \right) = $ ____甲____.

2. The equation of the plane tangent to the surface

$$z = x \cos y - y e^x$$

at the origin is ____乙____.

3. $\displaystyle\int_0^\pi \dfrac{1}{2 + \sin x} dx = $ ____丙____.

4. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the y-axis to generated a solid B. The volume of $B = $ ____丁____.

5. Let I be the interval of convergence of the series $\displaystyle\sum_{n=1}^\infty \dfrac{1}{n} \dfrac{(x+1)^n}{(x-3)^n}$. Then $I = $ ____戊____. (Note. The boundary points of I should also be considered.)

6. Let J be the area of the region bounded below by the x-axis and above by the curve parameterized by $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ for $0 \leq \theta \leq 2\pi$. Then $J = $ ____己____.

II. 計算與證明題（必須寫出演算證明過程）

1. (11%)
Sketch the graph of

$$r = \sin 4\theta, \ 0 \le \theta \le \pi$$

on the polar plane.

2. (11%)
Find the surface area of the solid D be obtained by revolving about the x-axis the region bounded by $y = x^{-2/3}$ and the x-axis to the right of $x = 1$; that is, $x \ge 1$.

3. (15%)
Evaluate $\iiint |xyz|\,dxdydz$ over the solid ellipsoid

$$E : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1 \ (a, b, c > 0).$$

4. (15%)
The plane $x + 2y + 3z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse.

Question. Find the points on the ellipse that lie closest to and farthest from the origin. (**Hint.** By the method of Lagrange multipliers.)