I. 填充題（共六題，每題八分，請將答案依甲、乙、丙....次序作答，不需演算過程）

1. $\lim_{x \to 0^+}(1 - \cos 2x)^{3x} = $ ____甲____.

2. Let $y = \int_x^{x^2} \sin(t^2)dt$. Then $\frac{d^2 y}{dx^2} = $ ____乙____.

3. $\int_0^1 \frac{\sqrt{2x - x^2}}{x}dx = $ ____丙____.

4. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the y-axis to generated a solid B. The volume of $B = $ ____丁____.

5. Let I be the interval of convergence of the series

$$\sum_{n=1}^{\infty}(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n})(1 - x)^n, \; x \in \mathbf{R}.$$

Then $I = $ ____戊____. (Note. The boundary points of I should also be considered.)

6. Let J be the area of the region bounded below by the x-axis and above by the curve parameterized by $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ for $0 \le \theta \le 2\pi$. Then $J = $ ____己____.

II. 計算與證明題（必須寫出演算證明過程）

1. (11%)
(a) Show that the equation $x^2 = \cos x$ has exactly two real roots.
(b) Let r_1 and r_2 be the roots of the equation. Find $r_1 + r_2$.

2. (11%)
Find the surface area of the solid D be obtained by revolving about the x-axis the region bounded by $y = x^{-2/3}$ and the x-axis to the right of $x = 1$; that is, $x \geq 1$.

3. (15%)
The curve $y = \sin x$, $0 \leq x \leq \pi$, is revolved about the line $y = c$, $0 \leq c \leq 1$, to generate the solid in the next figure.

(a) Find the value of c in $[0, 1]$ that minimizes the volume of the solid. What is the minimum value ?

(b) Find the value of c in $[0, 1]$ that maximizes the volume of the solid. What is the maximum value ?
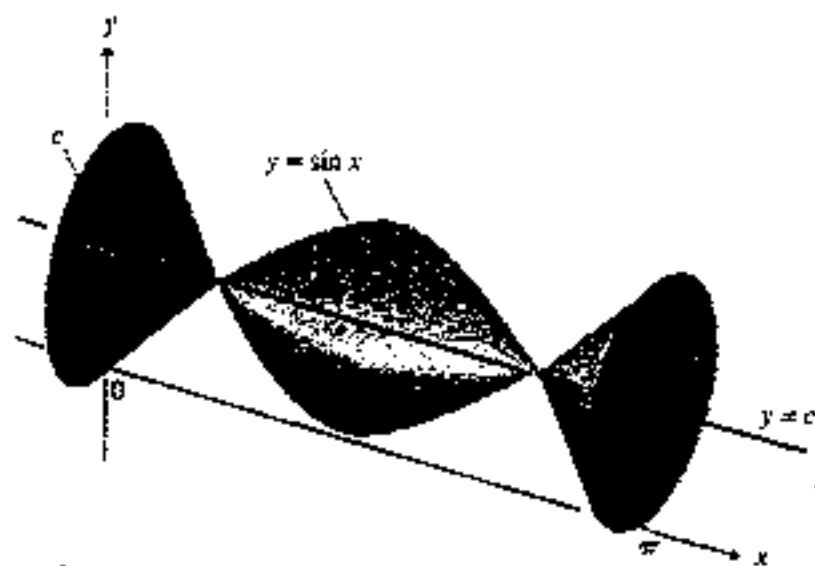
4. (15%)
The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse.
Question. Find the points on the ellipse that lie closest to and farthest from the origin. (**Hint.** By the method of Lagrange multipliers.)