國立清華大學命題紙

九十學年度 八系聯招 轉學生招生考試

科目 微積分 科號 003 共 1 頁 第 1 頁 *請在試卷【答案卷】內作答

I. 填充題(共五題,每題八分,請將答案依甲、乙、丙....次序作答,不需演算過程)

(1). Let $f(x) = \int_1^x t^{\frac{1}{3}} dt$, $x \in [1,5]$. Which of the following is true? Ans. 甲

a. $f(1) > 0$ b. $f(5) < 0$ c. $f(2) > f(4)$ d. $f(2) < f(4)$

(2). Let $a_1 \geq a_2 \geq \cdots \geq a_{10} > 0$. Find $\lim_{n\to\infty}(a_1^n + \cdots + a_{10}^n)^{1/n}$. Ans. 乙

(3). Evaluate the integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(\tan x)}{\sin x \cos x}\, dx$. Ans. 丙

(4). Evaluate the integral $\int_0^1 \int_x^1 xe^{2y^3}\, dydx$. Ans. 丁

(5). Find the area of the top half of the region inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = \cos\theta$. Ans. 戊

II. 計算與證明題(每題十二分,必須寫出演算證明過程)

(1). Let f be a C^1 function on $\mathbb{R}^3$. Show that if $w = f(r-s, s-t, t-r)$, then $\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 0$.

(2). Let f be a continuous real-valued function defined on $\mathbb{R}$. Using integration by parts, prove

$$\int_0^x \left(\int_0^t f(z)dz\right)dt = \int_0^x f(t)(x-t)dt.$$

(3). Let $\{a_n\}_{n=1}^{\infty}$ be a sequence with nonnegative terms. Suppose that $\sum_{n=1}^{\infty} a_n^2$ converges. Does $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converge? Prove or disprove it.

(4). Find $\int_D y^2 dA$, where D is the region bounded by the lines $x - 2y = 2$, $x - 2y = 5$, $2x + 3y = 1$ and $2x + 3y = 3$.

(5). Evaluate the line integral

$$\int_C (-y + e^x)dx + (x^3 + \sin y)dy,$$

where the curve $C = \{(x,y) \mid x^2 + y^2 = 1\}$ is traversed counterclockwise.